Day: Tuesday   Time: 02.30 PM TO 05.30 PM
Date: 14/05/2019   S-2019-2559   Max. Marks: 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Draw neat diagrams **WHEREVER** necessary.

Q.1 **a)** Prove by induction that sum of the cubes of 3 consecutive integers is divisible **(05)** by 9.

b) Construct truth tables to determine whether each of the following is a tautology **(05)** a contingency or a contradiction.
     i) $p \rightarrow (q \rightarrow p)$
     ii) $\left(p \wedge (\sim p \vee q)\right) \wedge \sim q$

OR

Q.1 **a)** Define Power set? If set X has 10 members, how many members do $P(x)$ has? **(05)** How many members of $P(x)$ are proper subset of x.

b) Among 50 students in a class, 26 got an A in the first examination and 21 got an **(05)** A in the second examination if 17 students did not get an A in either examination, how many students got A in both the examination.

Q.2 **a)** Consider the following relation R on A. **(05)**
$A = \{1,2,3\}$ and $R = \{(1,2), (2,3), (3,3)\}$
Find transitive closure of R.

b) Consider the set of words W = {Sheet, Last, Sky, Wash, Wind, Sit} **(05)**
Find W/R where R is the equivalence relation on W defined by either
    i)   "has the same number of letters as"
    ii)   "begins with same letters as"

OR

Q.2 **a)** Consider the Z of integers and an integer m > 1 we say that x is congruent to y **(05)** modulo m, written $x \equiv y \pmod{m}$ if $(x - y)$ is divisible by m. show that this defines an equivalence relation on Z.

b) Given A = {1,2,3,4} consider the following relation in A. **(05)**
$R = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$
    i)   Draw it's diagram
    ii)   Is R reflexive, symmetric, transitive or antisymmetric?

Q.3 **a)** Find the domain of real valued function. **(05)**
$$f(x) = \sqrt{81 - x^2}$$

b) Let f, g, h are functions on $X = \{1,2,3\}$: **(05)**
     $f = \{(1,2) \ (2,3) \ (3,1)\}$
     $g = \{(1,2) \ (2,1) \ (3,3)\}$ Compute Find $f \circ g$, $g \circ f$ and $f \circ g \circ h$
     $h = \{(1,1) \ (2,2) \ (3,1)\}$

OR

Q.3 **a)** Define function and give types of function. Let the functions f and g defined by **(05)**
f (x) = 2x + 1 & g (x) = x² − 2 . Find the formula defining the composition
function g ∘ f.

b) Solve the following recursive function using substitution: **(05)**

$$f(n) = f\left(\frac{n}{2}\right) + 1, \quad f(1) = 1 \text{ for n to be integer greater than or equal to 1.}$$

Q.4 **a)** Explain Breadth first search traversal with example. **(05)**

b) Draw all trees with exactly six vertices. **(05)**

OR

Q.4 **a)** Which connected graphs can be both regular and bipartite? **(05)**

b) Write the Kruskal's algorithm and give example. **(05)**

Q.5 **a)** What do you mean by homomorphism of semigroups? Give example. **(05)**

b) Define the terms with example: **i)** Monoid **ii)** Group **(05)**

OR

Q.5 **a)** Consider the set Q of rational number, let * be operation on Q defined by **(05)**
a * b = a + b − ab.
i) Find 3 * 4, 2 * (-5) & 7 * $\frac{1}{2}$

ii) Find the identity element for *.

b) Let S be a semigroup with identity of let b and b' be inverses of each other **(05)**
show that b = b' that is that inverses are unique if they exist.

Q.6 **a)** Find the probability of getting a total 7, atleast once in 3 tosses of a pair of fair **(05)**
dice.

b) A box contains 6 white balls and 5 black balls find the number of ways 4 balls **(05)**
can be drawn from the box if :
i) Two must be white,
ii) All of them must have the same colour.

OR

Q.6 **a)** Suppose A and B are events with P(A) = 0.6, P(B) = 0.3 and P(A ∩ B) = 0.2 **(05)**
find the probability that:
i) A does not occur
ii) B does not occur
iii) A or B occurs

b) Suppose repetitions are not permitted how many three digit numbers can be **(05)**
formed from six digits 2, 3, 5, 6, 7, and 9 ?

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