M. Tech.-I (Civil-Hydraulic Engineering) (CBCS – 2015 Course) : SUMMER - 2019

SUBJECT: COMPUTATIONAL METHODS IN HYDRAULIC ENGINEERING

Day : Tuesday

Time : 11.00 AM TO 02.00 PM

Date : 21/05/2019

S-2019-3371

Max. Marks: 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate FULL marks.
- 3) Answer to both the sections should be written in **SAME** Answer book.
- 4) Use of non-programmable calculator is allowed.

SECTION - I

Q.1 a) If f(z) is an analytic function with constant modulus, show that f(z) is constant. (05)

b) Find analytic function
$$f(z) = u(r, \theta) + iv(r, \theta)$$
 such that $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$. (05)

OR

a) Show that the polar form of Cauchy-Riemann equation are
$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$$
 (05)

b) Determine the analytic function whose real part is
$$\frac{y}{x^2 + y^2}$$
. (05)

- Q.2 a) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in the z-plane is (05) mapped into the circle |w|=1. Which portion of the z-plane corresponds to the interior of the circle.
 - b) Find the transformation which maps the semi-infinite strip of width π (05) bounded by the lines $v = 0, v = \pi$ and u = 0 into the upper half of the z plane.

OR

a) Show that under the transformation $w = \frac{1}{z}$, circle $x^2 + y^2 - 6x = 0$ is (05) transformed into a straight line in the w-plane.

b) Evaluate:
$$\int_{C} \frac{e^{2z}}{(z+1)^4} dz$$
, where C is $|z| = 4$. (05)

P.T.O.

Q.3 Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of figure with (10) boundary values as shown.

0	5	00 1	000 50	00	- 0
0		u ₁	u ₂	u ₃	$\frac{0}{1000}$
1000	A	u4	U5	u_6B	2000
2000		u ₇	u ₈	u9	1000
1000	-				1000
0	50	0 10	00 50	00	- 0

OR

a) Classify the following equations:

i)
$$u_{xx} + 4u_{xy} - u_x + 2u_y = 0$$
.

ii)
$$x^2 u_{xx} + (1 - y^2) u_{yy} = 0, -\infty < x < \infty, -1 < y < 1$$

b) Solve by Relaxation method, the equations: (06)

$$9x - 2y + z = 50$$

$$x + 5y - 3z = 18$$

$$-2x + 2y + 7z = 19$$

SECTION - II

Q.4 a) Fit a straight line y = a + bx to the following data by method of least (04) squares.

x:	0	1	3	6	8
y:	1	3	2	5	4

b) Evaluate $\int_{0}^{6} \frac{dx}{1+x^2}$ by using i) Simpson's $1/3^{\text{rd}}$ rule ii) Simpson's 3/8 rule. (06)

OR

a) Fit a second degree parabola to the following data:

X:	1929	1930	1931	1932	1933	1934	1935	1936	1937
у:	352	356	357	358	360	361	361	360	359

b) Use Trapezoidal rule to evaluate $\int_{0}^{1} x^{3} dx$ considering five sub-intervals. (05)

Q.5 a) Calculate the first four moments of the following distribution about the (06) mean.

x:	0	1	2	3	4	5	6	7	8
f:	1	8	28	56	70	56	28	8	1

Also evaluate β_1 and β_2 .

b) If
$$r_{12} = 0.25$$
, $r_{13} = 0.35$, and $r_{23} = 0.45$, then find $R_{2.13}$. (04)

OR

- a) The two regression equations of the variables x and y are x = 19.13 0.87y (0) and y = 11.64 0.50x find i) mean of x's and y's. ii) Correlation coefficient between x and y.
- b) Ten participants in a contest are ranked by two judges as follows: (05)

x:	1	6	5	10	3	2	4	9	7	8
y:	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient.

Q.6 a) Three machinesM₁,M₂ and M₃ produce identical items. Of their respective output 5% and 4% and 3% of items are faulty. On a certain day M₁ has produced 25% of the total output, M₂ has produced 30% and M₃ the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output?

b) A random variable x has the following probability function. (05)

x:	0	1	2	3	4	5	6	7
P(x):	0	k	2k	2k	3k	k ²	$2k^2$	$7k^2+k$

i) Find value of k ii) Evaluate $p(x < 6), p(x \ge 6)$

OR

- a) If the probability of a bad reaction from a certain injection is 0.001, (05) determine the chance that out of 2,000 individuals more than two will get bad reaction.
- b) A die was thrown 60 times and the following frequency distribution was (05) observed:

	faces:	1	2	3	4	5	6
,	f_0 :	5	6	4	7	11	7

Test whether the die is unbiased