# S.Y.B.SC. SEM – IV (2014 Course) : SUMMER - 2019
## SUBJECT : MATHEMATICS : VECTOR CALCULUS (M-41)

Day : Wednesday
Date : 08/05/2019

**S-2019-0986**

Time 03.00 PM TO 05.00 PM
Max. Marks : 40

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**N.B.**

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

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**Q.1** Attempt any **TWO** of the following: **(10)**

**a)** Prove that a non-constant vector function $\bar{u}(t)$ is of constant direction if and only if $\bar{u}\times\frac{d\bar{u}}{dt}=\bar{0}$ .

**b)** If $\bar{r}=x\cos y\hat{i}+x\sin y\hat{j}+c\log\left[x+\sqrt{x^2-c^2}\right]\hat{k}$, find the unit vector perpendicular to both $\frac{\partial\bar{r}}{\partial x}$ and $\frac{\partial\bar{r}}{\partial y}$ such that $\frac{\partial\bar{r}}{\partial x}$, $\frac{\partial\bar{r}}{\partial y}$ and unit vector form a right handed system.

**c)** Find the angle between the surfaces $x^2y+z=3$ and $x\log z-y^2+4=0$ at the point (-1, 2, 1).

**Q.2** Attempt any **TWO** of the following: **(10)**

**a)** If $\bar{u}$ is a vector point function and $\phi$ is a scalar point function then show that $\nabla\bullet(\phi\bar{u})=(\nabla\phi)\bullet\bar{u}+\phi(\nabla\bullet\bar{u})$ .

**b)** Find the directional derivative of $x^2y+xz^2-2$ at $A(1,1,-1)$ along $\overrightarrow{AB}$, where $B(2,-1,3)$.

**c)** If $\bar{f}=z\hat{i}+x\hat{j}-3y^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$, evaluate $\iint_S \bar{f}\bullet\bar{n}\,dS$ .

**Q.3** Attempt any **TWO** of the following: **(10)**

**a)** Let $\bar{f}$ be a continuously differentiable vector field on a region R. Then show that $\bar{f}$ is conservative if and only if it is the gradient of some scalar point function $\phi$ defined on R.

**b)** Using Green's theorem evaluate $\oint_C[(y-\sin x)dx+\cos x\,dy]$, where C is perimeter of the triangle with vertices $O(0,0), A\left(\frac{\pi}{2},0\right)$ and $B\left(\frac{\pi}{2},1\right)$.

**c)** Prove by using Stoke's theorem that $\int_C(\sin z\,dx-\cos x\,dy+\sin y\,dz)=2,$ where C is the boundary of the rectangle $0\le x\le\pi,\ 0\le y\le 1;\ z=3$.

P.T.O.

**Q.4** Attempt any **FIVE** of the following: **(10)**

**a)** If $\bar{r} = (t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k}$ then find $\left|\frac{d^2\bar{r}}{dt^2}\right|$ at $t = 2$.

**b)** Find the equations of the normal line to the surface $xy + yz + zx = 7$ at $(1,1,3)$

**c)** Prove that $\nabla \bullet \left(r^n \bar{r}\right) = (3+n)r^n$ .

**d)** Prove that curl $(grad\,\phi) = \bar{0}$ .

**e)** If $\bar{f}(t) = (t - t^2)\hat{i} + 2t^3\hat{j} - 3\hat{k}$; find $\int_1^2 \bar{f}(t)\,dt$ .

**f)** Define gradient of scalar point function.

**g)** State Gauss's Divergence theorem.

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