F.Y.B.Sc. SEM – II (CBCS 2018 COURSE) : SUMMER - 2019

SUBJECT: MATHEMATICS : INTEGRAL CALCULUS & DIFFERENTIAL EQUATIONS

Day : Thursday
Date : 02/05/2019

S-2019-0795

Time : 11.00 A.M TO 02.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 A) Select the correct alternatives of the following: **[06]**

i) $\int_0^{\pi/2} \sin^4 \theta d\theta =$ _______.

a) $\frac{3\pi}{128}$ **b)** $\frac{3\pi}{64}$ **c)** $\frac{3\pi}{32}$ **d)** $\frac{3\pi}{16}$

ii) $\int_0^{\pi/2} \sin^4 \theta \cos^2 \theta d\theta =$ ________.

a) $\frac{5\pi}{256}$ **b)** $\frac{3\pi}{128}$ **c)** $\frac{3\pi}{256}$ **d)** $\frac{5\pi}{128}$

iii) $\int \frac{dx}{x^2-1} =$ ________.

a) $\log\left(\frac{x-1}{x+1}\right)+C$ **c)** $\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)+C$

b) $\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+C$ **d)** $\frac{1}{\sqrt{2}}\log\left(\frac{x-1}{x+1}\right)+C$

iv) Degree of the differential equation $L\frac{d^2Q}{dt^2}+\frac{1}{C}$ Q=E $Sin\,wt$, is ______.

a) 1 **b)** 2 **c)** 3 **d)** none of these

v) Substitution for solving the differential equation $\frac{dy}{dx}=\frac{3y+2x+4}{4x+6y+4}$, is ______.

a) $u=x+y$ **b)** $u=2x+3y$ **c)** $u=3x+2y$ **d)** $u=4x+6y$

vi) $y=ax+b$ is an equation where a and b are arbitrary constants then its differential equation is _______.

a) $\frac{d^2y}{dx^2}+y=0$ **b)** $\frac{dy}{dx}=0$ **c)** $\frac{d^2y}{dx^2}=0$ **d)** $\frac{d^2y}{dx^2}+xy=0$

B) Solve the following: **[06]**

i) State the formula for obtaining surface area of the curve $r=f(\theta)$.

ii) For evaluating $\int\frac{dx}{9+6\sin x}$, what is the substitution?

iii) Evaluate : $\int_0^{\pi/2} \sin^5 x\cos^6 x dx$.

P.T.O.

iv) Define Bernoulli's differential equation.

v) Define integrating factor of the differential equation.

vi) Form the differential equation of $y = cx - 2c + c^3$ where c is arbitrary constant.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) Evaluate : $\int \frac{x^2+1}{x^4+1}\, dx.$

b) Evaluate : $\int \frac{x^2+1}{(2x+1)(x^2-1)}\, dx.$

c) Evaluate : $\int \frac{(x-8)\,dx}{(2x-1)(x^2+x+3)}.$

d) Find the orthogonal trajectories of the family of curves given by $y = ce^{-2x}$, where c is parameter.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) Define homogeneous differential equation and explain the method of its solution.

b) Solve the differential equation $(x^2+y^2)dx = 2xy\, dy.$

c) Solve the differential equation $(x^2+y^2-a^2)x\,dx + (x^2-y^2-b^2)y\, dy = 0.$

d) Evaluate : $\int_0^{\pi/4} 4\cos^4 x \sin^4 x\, dx.$

e) Evaluate : $\int \operatorname{cosec}^4 x\, dx.$

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Prove that the necessary and sufficient condition for the equation $Mdx + Ndy = 0$, where M and N are functions of x and y, to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$

b) Solve : $(x^3+xy^4)\, dx + 2y^3\, dy = 0.$

c) Solve : $3\frac{dy}{dx} + \frac{2}{x+1}\, y = \frac{x^3}{y^2}.$

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) Evaluate $\int \frac{dx}{a+b\cos x}$ if **i)** $a > b$ and **ii)** $a < b$.

b) The area bounded by the hyperbola $xy = 4$ and the line $x + y = 5$ is revolved about the x–axis. Find the volume of the solid thus generated.

c) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ about the line $y = 0$.

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