S.Y.B.SC. SEM – IV (2014 Course) : SUMMER - 2019

SUBJECT : MATHEMATICS : COMPLEX VARIABLES (M – 42)

Day : Thursday
Date : 25/04/2019

S-2019-0988

Time : 03.00 PM TO 05.00 PM
Max. Marks : 40

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

a) Show that the real and the imaginary parts of an analytic function f(z) = u + iv, satisfy Laplace's differential equation.

b) Find an analytic function of which e^x (xcosy – ysiny) is the real part.

c) If f(z) is an analytic function, then prove that $\nabla^2 \left[Rf(z) \right]^2 = 2\left|f'(z)\right|^2$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

a) Evaluate $\int_C \frac{e^z dz}{z}$, where C is the unit circle with centre at 0, using Cauchy's integral formula. Hence or otherwise show that $\int_0^{2\pi} e^{\cos\theta} . \cos(\sin\theta)\, d\theta = 2\pi$.

b) Using Cauchy's integral formula evaluate $\int_C \frac{dz}{z^3(z+4)}$, where C is the circle $|z| = 2$.

c) Obtain the expansion of $\frac{3z-3}{(2z-3)(z-2)}$.

Q.3 Attempt **ANY TWO** of the following: **[10]**

a) State and prove Cauchy's residue theorem.

b) Evaluate by contour integration $\int_C \frac{3z^2+2}{(z-1)(z^2+9)}$ dz, where C is a circle $|z-2| = 2$.

c) By using Cauchy's residue theorem evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\, dx$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

a) Obtain Maclaurin's series for sin z.

b) Evaluate $\lim_{z\to 1+i} \frac{z^4+4}{z-(1+i)}$.

c) Show that a function $f(z) = \bar{z}$ is continuous everywhere but differentiable nowhere.

d) Determine the poles and their order for the function :

$$f(z) = \frac{z^3+1}{(z^2+2)(z^2-1)^3}$$

e) Find the residues of $f(z) = \frac{z^2}{(z-2)^2(z+3)}$ at the simple poles.

f) State Cauchy's theorem of complex variables.

g) Evaluate $\int_0^{1+i} (x - y + ix^2)\, dz$, along the straight line z = 0 to z = 1 + i.

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