F.Y.B.SC. SEM – II (CBCS - 2016 Course) : SUMMER - 2019
SUBJECT : MATHEMATICS : INTEGRAL CALCULUS & DIFFERENTIAL EQUATIONS

Day : Tuedsay
Date : 07/05/2019

Time : 03.00 P.M. To 06.00 P.M
Max. Marks :60

S-2019-0824

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q. 1 A) Select the correct alternatives of the following: **(06)**

i) $\int_0^{\pi/2} \sin^7 \theta d\theta =$ ______

a) $\frac{32}{105}$ b) $\frac{64}{35}$ c) $\frac{8}{15}$ d) $\frac{16}{35}$

ii) $\int_0^{\pi/2} \sin^4 \theta \cos^2 \theta d\theta =$ _____

a) $\frac{5\pi}{128}$ b) $\frac{\pi}{64}$ c) $\frac{\pi}{32}$ d) $\frac{\pi}{16}$

iii) $\int \sqrt{a^2 - x^2 dx} =$ _____

a) $\sin^{-1}\left(\frac{x}{a}\right)+c$ b) $\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

c) $\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\log\left[x+\sqrt{a^2-x^2}\right]+c$ d) None of these

iv) If the differential equation $Mdx + Ndy = 0$, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the differential equation is _____

a) Exact b) linear c) non-linear d) homogenous

v) Integrating factor of the following equation $Mdx + Ndy = 0,\ f_1(xy)ydx + f_2(xy)xdy = 0$ is _____.

a) $\frac{1}{Mx+Ny}$ $if\ Mx+Ny \neq 0$ b) $\frac{1}{Nx+My}$ $if\ Nx+My \neq 0$

c) $\frac{1}{Mx-Ny}$ $if\ Mx-Ny \neq 0$ d) $\frac{1}{Nx-My}$ $if\ Nx-My \neq 0$

vi) Degree of the differential equation $2y = \frac{ax}{\left(\frac{dy}{dx}\right)} + x\frac{dy}{dx}$ *is* _____

a) 2 b) 3 c) 1 d) 0

B) Solve the following: **(06)**

i) State the formula for obtaining surface area of the curve $y = f(x)$

ii) For evaluating $\int \frac{dx}{4+5\cos x}$, which is the substitution?

iii) Evaluate $\int_0^{\pi/2} \sin^4 x \cos^4 x\, dx$

(PTO)

iv) Form the differential equation of $y = ae^{-2x} + be^{2x}$, where a & b are arbitrary constants.

v) Define Bernoulli's differential equation.

vi) Convert the differential equation $3\frac{dy}{dx} + \frac{2}{x+1}y = \frac{x^3}{y^2}$ into linear differential equation of first order and first degree.

Q.2 Attempt **ANY THREE** of the following: **(12)**

a) Evaluate $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx$

b) Evaluate $\int \frac{x^2+2}{x^3-1} dx$

c) Evaluate $\int \frac{x^2+1}{x^4-1} dx$

d) Find the orthogonal trajectories of the family of rectangular hyperbolas $xy = c^2$

Q.3 Attempt **ANY FOUR** of the following: **(12)**

a) Show that $\int \cos ec^n x\, dx = \frac{\cos ec^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} \int \cos ec^{n-2} x\, dx$

b) Evaluate $\int \tan^5 x\, dx$

c) Evaluate $\int_0^{\pi/4} \cos^3 2x \sin^4 4x\, dx$

d) Solve $(1 + xy^2)dx + (1 + x^2 y)dy = 0$

e) Solve the differential equation $yz\, dx + 2xz\, dy - 3xy\, dz = 0$

Q.4 Attempt **ANY TWO** of the following: **(12)**

a) Prove that the solution of the differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone is $ye^{\int p(x)dx} = \int e^{\int p(x)dx} q(x)\, dx + c$

b) Solve : $y(xy+1)\, dx + x(1 + xy + x^2y^2)\, dy = 0$

c) Solve : $\frac{dy}{dx} - y \tan x + y^2 \sec^2 x = 0$

Q.5 Attempt **ANY TWO** of the following: **(12)**

a) Evaluate $\int \frac{dx}{a + b\sin x}$ if (i) $a^2 > b^2$ (ii) $a^2 < b^2$

b) Find the area of surface of revolution generated by revolving about $X-axis$, the arc of the parabola $y^2 = 12x$ from $x = 0$ to $x = 3$

c) Show that length of arc of the curve $x = 1 - \cos t + \frac{3}{5}t, y = \frac{4}{5}\sin t, t = 0\ to\ t = \pi$ is $\pi + \frac{6}{5}$

* * * * *