S.Y.B.SC. SEM – IV (CBCS - 2016 Course) : SUMMER - 2019
SUBJECT: MATHEMATICS: VECTOR CALCULUS

Day: Saturday
Date: 11/05/2019

S-2019-0849

Time: 11.00 A.M. To 02.00 P.M.
Max. Marks: 60

N.B:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) A differentiable vector function $\bar{u}(t)$ on [a,b] is of constant magnitude if and only if $\bar{u}.\frac{d\bar{u}}{dt}=0, \forall t \in [a,b]$.

b) If $\bar{u}=5t^2\bar{i}+t\bar{j}-t^3\bar{k}$ *and* $\bar{v}=\sin t\bar{i}-\cos t\bar{j}$ *find*

$i)\ \frac{d}{dt}\left(\bar{u}\times\bar{v}\right) \quad ii)\ \frac{d}{dt}\left(\bar{v}.\bar{v}\right) \quad iii)\ \frac{d}{dt}\left(\bar{u}.\bar{v}\right)$ *at* $t=2$.

c) Find the acute angle between the tangents to the curve $\bar{r}=t^2\bar{i}-2t\bar{j}+t^3\bar{k}$ at the points $t=1$ and $t=2$.

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) If *If* $\phi = xy+yz+xz$ *and* $\bar{u}=x^2\bar{i}+y^2\bar{j}+z^2\bar{k}$ *then find* $\frac{\partial^2\left(\phi\bar{u}\right)}{\partial z\partial x}$ *at* $(3,2,-4)$.

b) If $\bar{r}=x\cos y\bar{i}+y\sin y\bar{j}+ae^{my}\bar{k}$, find : $\frac{\frac{\partial\bar{r}}{\partial x}\times\frac{\partial\bar{r}}{\partial y}}{\left|\frac{\partial\bar{r}}{\partial x}\times\frac{\partial\bar{r}}{ry}\right|}$.

c) Find the scalar function $\phi(x,y,z)$ if grand $\phi = y(2xz-1)\bar{i}+x(xz-1)\bar{j}+(x^2y+4)\bar{k}$ *and* $\phi(2,1,-1)=0$.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) Verify Green's theorem in the plane $\oint(y-\sin x)dx+\cos x dy$ where C is the perimeter of the triangle with vertices $0(0,0), A\left(\frac{\pi}{2},0\right)$ and $B\left(\frac{\pi}{2},1\right)$.

b) Evaluate $\iint_S \bar{f}.\bar{n}\,ds$ where $\bar{f}=4xz\bar{i}-y^2\bar{j}+yz\bar{k}$ *and* S is the surface bounded by $2x+2y+2z=6, x=0, y=0, z=0$.

c) Evaluate $\int_C\left[\left(x^2-y^2\right)\bar{i}+2xy\bar{j}\right]d\bar{r}$ around a rectangle with vertices at (0, 0), (a,0), (a,b) and (0,b) traversed in contour clockwise direction.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) If $\bar{u}$ is a vector point function and ϕ is a scalar point function then show that $\nabla.\left(\phi\bar{u}\right)=\left(\nabla\phi\right).\bar{u}+\phi\left(\nabla.\bar{u}\right)$.

b) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point P(2, -1, 1) in the direction of $\bar{i}+2\bar{j}+2\bar{k}$.

c) Find the angle between the surfaces $x^2y+z=3$ and $x\log z - y^2+4=0$, at the point (-1, 2, 1).

d) *If* $\bar{r}=x\bar{i}+y\bar{i}+z\bar{k}$ *and* $r=\left|\bar{r}\right|=\sqrt{x^2+y^2+z^2}$ *find,*

i) ∇r ii) $div\,\bar{r}$

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) Eliminate $\bar{a}$ and $\bar{b}$ from $\bar{r}=\bar{a}\cos 2t+\bar{b}\sin 2t$ and obtain the differential equation.

b) Find the unit vector normal to the surface $x^2+y^2-z=1$ at $(1,1,1)$.

c) Define gradient of a scalar point function and divergence of a vector point function.

d) Find maximum value of directional derivative of $\phi=xy^2+yz^3$ at the point P(2, -1, 1).

e) If $\bar{f}(t)=\left(t-t^2\right)\bar{i}+2t^3\bar{j}-3\bar{k}$, *find* $\int_1^2 \bar{f}(t)dt$.

f) Show that vector point function $\bar{v}=x^2\bar{i}+y^2\bar{j}+z^2\bar{k}$ is irrotational.

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