F.Y.B.Sc. SEM – I (CBCS 2018 COURSE) : SUMMER - 2019

SUBJECT : MATHEMATICS : CALCULUS

Day : Tuesday
Date : 07/05/2019

Time : 03.00 PM TO 06.00 PM
Max. marks : 60

S-2019-0780

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following. **(12)**

a) Prove that every continuous function on closed and bounded interval attains its bounds.

b) If the function defined below is continuous in given domain, then find the values of a and b, where

$$f(x) = ax + b \quad , \quad 0 \le x < 2$$
$$= bx + 11 \quad , \quad 2 \le x \le 4$$

and $f(3) = 2$.

c) If $y = a\cos(\log x) + b\sin(\log x)$,then show that

$x^2 y_{n+2} + (2n+1)\,x\,y_{n+1} + (n^2+1)\,y_n = 0$.

Q.2 Attempt **ANY TWO** of the following. **(12)**

a) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$.

b) Show that a sequence $\{a_n\}$ is monotonic and bounded, where

$$a_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \frac{1}{3^3+1} + \ldots\ldots + \frac{1}{3^n+1} \ .$$

c) Discuss the convergence of series $\sum \frac{x^n}{n(n+1)}$.

Q.3 Attempt **ANY TWO** of the following. **(12)**

a) State and prove Rolle's mean value theorem.

b) Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$ over [2,5]. Find the value of c and θ .

c) If in the Cauchy's mean value theorem on [a,b], we write $f(x) = e^x$ and $g(x) = e^{-x}$ then show that 'c' is the arithematic mean between a and b .

Q.4 Attempt **ANY THREE** of the following. **(12)**

a) Discuss the continuity of the function f(x) , if $f(x) = \sqrt{\frac{x-1}{x+3}}$.

b) Evaluate : $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$.

c) Discuss the convergence of given series by using ratio test $\sum \frac{n}{2^n}$.

d) If $y = \log(ax+b)$, find y_n .

P.T.O.

Q.5 Attempt **ANY FOUR** of the following. **(12)**

a) Define : i) suprimum
ii) infimum.

b) Discuss the continuity of the function $f(x)=\sqrt{(x-2)(x-4)}$.

c) Define : i) bounded sequence
ii) convergent sequence.

d) Find the expansion of tan x upto the terms in x^5 .

e) Evaluate : $\lim_{x\to 0} x\log x$.

f) If y = cos (ax+b) then find y_n .

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