S.Y.B.SC. SEM – IV (CBCS - 2016 Course) : SUMMER - 2019
SUBJECT: MATHEMATICS: COMPLEX VARIABLES

Day : Thursday
Date : 25/04/2019

S-2019-0851

Time: 11.00 A.M. To 02.00 P.M.
Max. Marks: 60.

N.B.:
1) All questions are **COMPULSORY.**
2) Figures to the **RIGHT** indicate full marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Show that a necessary condition that a function w = f(z) = u(x, y) + i v(x, y) be analytic at a point z = x + iy of its domain D is that at (x, y), the first order partial derivatives of u and v w.r.t. x and y exist and satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.

b) Find the analytic function whose imaginary part is $4xy - x^3 + 3xy^2$.

c) Evaluate: $\int_C \frac{ze^z}{(z-1)^3} dz$, where C is | z–1 | = 2.

Q.2 Attempt any **TWO** of the following: **(12)**

a) Using Cauchy's theorem obtain the value of $\int_C e^z dz$, where C is the circle $|z|=1$ and deduce that $\int_0^{2\pi} e^{\cos\theta}\left[\sin(\theta+\sin\theta)\right]d\theta = 0$.

b) Evaluate: $\int_C \frac{z+6}{z^2-4} dz$, where
i) C is the circle $|z|=1$
ii) C is the circle $|z-2|=1$
iii) C is the circle $|z+2|=1$

c) Evaluate by contour integration:
$\int_C \frac{5z-2}{z(z-1)} dz$, where C is the circle | z | = 3 taken counter clockwise.

Q.3 Attempt any **TWO** of the following: **(12)**

a) State and prove Cauchy's residue theorem.

b) By contour integration, evaluate $\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$.

c) If f(z) and $\overline{f(z)}$ both are analytic functions of z, then prove that f(z) is constant.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Evaluate: $\lim_{z \to e^{i\pi/4}} \frac{z^2}{z^4 + z^2 + 1}$.

b) Evaluate $\int_C \frac{z^3}{z-2i} dz$, where C is the circle $|z-2|=5$, by using Cauchy's integral formula.

c) Obtain the Laurent's series of the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$.

d) Prove that the sum of the residues of the function $\frac{e^z}{z^2+a^2}$ is $\frac{\sin a}{a}$

P.T.O.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Evaluate: $\lim\limits_{z\to(1+i)} \dfrac{z^2-z+1-i}{z^2-2z+2}$.

b) If a function f of a complex variable z is differentiable at z_0, then it is continuous at z_0.

c) Evaluate: $\int_C (x^2+y^2-xyi)\,dz$, where C is the line segment from z = 0 to z = 1 + i.

d) Find the zeros of $(z^4+8z^2+16)(z^2+z+1)$.

e) Define pole of the function with an example.

f) Find the residue of $f(z)=\dfrac{z}{(z-2)(z+3)}$ at simple pole z = 2.

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