F.Y.B.SC. SEM – I (CBCS - 2016 Course) : SUMMER - 2019

SUBJECT : MATHEMATICS : CALCULUS

Day : Thursday
Date : 02/05/2019

S-2019-0810

Time : 11.00 A.M TO 02.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 A) Choose the correct alternatives of the following: **[06]**

i) $\lim_{x\to 0} \dfrac{\log \sin x}{\cot x} =$ ________.

a) $\frac{1}{2}$ **b)** $\frac{3}{2}$ **c)** -1 **d)** 0

ii) If $y = e^{ax}$ then $y_n =$ _______.

a) $a^n e^{ax}$ **b)** ae^{ax} **c)** $a^{n+1}e^{ax}$ **d)** e^{ax}

iii) If $y = \dfrac{1}{3x+4}$ then $y_n =$ _______.

a) $\dfrac{(-1)^n n!3^n}{(3x+4)^n}$ **b)** $\dfrac{n!3^n}{(3x+4)^n}$ **c)** $\dfrac{n!(3)^n}{(3x+4)^{n+1}}$ **d)** $\dfrac{(-1)^n n!3^n}{(3x+4)^{n+1}}$

iv) A series $\sum_{n=1}^{\infty} \dfrac{n}{2^n}$ is _________.

a) divergent **b)** oscillatory **c)** convergent **d)** none of these

v) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots\ldots\ldots\ldots\ldots \quad \forall x \in R$ is an expansion of_________.

a) $\frac{1}{x}$ **b)** $\sin x$ **c)** $\cos x$ **d)** e^x

vi) Every monotonic sequence is always ________.

a) bounded below **c)** either (a) or (b)
b) bounded above **d)** both (a) and (b)

B) Answer the following: **[06]**

i) State geometrical meaning of Lagrange's mean value theorem.

ii) If $y = x^3 \cos x$, then find y_n.

iii) State Heine's property.

iv) Evaluate : $\lim_{x\to 0}\left[\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right]$.

v) Using Maclaurin's theorem, write the expansion of e^x.

vi) Define supremum of a function.

P.T.O.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) State and prove Cauchy's mean value theorem.

b) Using Lagrange's mean value theorem, prove that

$$\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4}+\frac{1}{6}.$$

c) Verify Rolle's theorem for the function $f(x)=\log\left[\frac{x^2+ab}{(a+b)x}\right]$ over [a, b] where a > 0.

d) Show that $\{a_n\}$ where $a_n=\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\dots\dots\dots+\frac{1}{2n}$ is bounded.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) Prove that every continuous function on closed and bounded interval is bounded.

b) Discuss the continuity of $f(x)=\sqrt{\frac{x-1}{x+3}}$.

c) Evaluate $\lim_{x\to 1}\left(\frac{1+\log x-x}{1-2x+x^2}\right)$.

d) Show that the function f defined by f(x) = |x| is continuous but not differentiable at x = 0.

e) Verify L.M.V.T. for f(x) = $\log x$ on [1, e].

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if p > 1.

b) Show that a sequence $\{S_n\}$ where $S_n=\left(1+\frac{1}{n}\right)^n$ is monotonic and bounded.

c) Using Maclaurin's theorem, prove that

$$\log\sec x=\frac{x^2}{2!}+\frac{2x^4}{4!}+\frac{16x^6}{6!}+\dots\dots\dots$$

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) State and prove that Leibnitz's theorem for n^{th} derivative of the product of two functions of x.

b) If y = a cos($\log x$) + b sin($\log x$) then show that
$x^2 y_{n+2}+(2n+1)\,x\,y_{n+1}+(n^2+1)\,y_n=0$.

c) Find the value of c and θ in the conclusion of L.M.V.T. for the function
f(x) = x^2-2x+3 over [1, 1.5].

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