Day : Friday   Time : 12.00 NOON TO 02.00 PM
Date : 03/05/2019   **S-2019-0948**   Max. Marks : 40

N.B.:
    1)   All questions are **COMPULSORY**.
    2)   Figures to the right indicate **FULL** marks.

Q.1   Attempt **ANY TWO** of the following: **[10]**

    a)   Prove that every continuous function on closed and bounded interval is bounded.

    b)   Discuss the continuity of $f(x) = \sqrt{\dfrac{x-2}{x+4}}$.

    c)   If $y = a\cos(logx) + b\sin(logx)$, then show that
$$x^2 y_{n+2} + (2n+1) xy_{n+1} + (n^2 +1) y_n = 0 .$$

Q.2   Attempt **ANY TWO** of the following: **[10]**

    a)   State and prove Rolle's mean value theorem.

    b)   Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$, over [2, 5]. Find the value of c and θ.

    c)   Verify Cauchy's mean value theorem for the functions $f(x) = e^x$ and $g(x) = x$ over [0, 1].

Q.3   Attempt **ANY TWO** of the following: **[10]**

    a)   Show that a sequence {S_n} where $S_n = \left(1 + \dfrac{1}{n}\right)^n$ is monotonic and bounded.

    b)   Discuss the convergence of $\displaystyle\sum_{n=1}^{\infty} \dfrac{x^n}{n(n+1)}$.

    c)   Using Taylor's theorem prove that $e^x \cos x = 1 + x - \dfrac{x^3}{3} - \dfrac{x^4}{6} - \dfrac{x^5}{30} + \ldots\ldots$

Q.4   Attempt **ANY FIVE** of the following: **[10]**

    a)   Define supremum and infinium of a function.

    b)   Show that the function f defined by $f(x) = |x|$ is continuous but not differentiable.

    c)   Evaluate : $\displaystyle\lim_{x \to \frac{\pi}{2}} \dfrac{\log(x - \pi/2)}{\tan x}$.

    d)   If $y = x^2 e^{2x}$, find y_n.

    e)   If $y = \dfrac{1}{3x+5} + e^{5x} + \log(3 - 2x) + 3^{4x}$, find y_n .

    f)   Discuss the convergence of sequence {a_n} where $a_n = 3n - 4$.

    g)   Discuss the convergence of $\displaystyle\sum \dfrac{1}{n!}$.

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