F.Y.B.Sc. SEM – I (CBCS 2018 COURSE) : SUMMER - 2019
SUBJECT : MATHEMATICS : ALGEBRA

Day : Wednesday
Date : 24/04/2019

Time : 03.00 PM TO 06.00 PM
Max. Marks : 60

S-2019-0778

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) A necessary and sufficient condition for a square matrix A to have the inverse is that A is non-singular i.e., $|A| \neq 0$.

b) Find the non-singular matrices P and Q such that PAQ is the normal form and find rank of A, where $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$.

c) Verify Cayley – Hamilton theorem for the matrix,

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) If a and b are any two integers with $a \neq 0$ then there exist unique integers q and r such that $b = aq + r$, where $0 \leq r < |a|$.

b) Find the g.c.d. of 3587 and 1819 and express it in the form 3587m + 1819n; find the values of m and n.

c) If z_1 and z_2 are any two complex numbers then show that:

i) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ **ii)** $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) State and prove De Moivre's theorem for positive and negative integers.

b) If $\sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$ then prove that,
$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$,
$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$.

c) Let $a, b, c, d, x, y \in \mathbb{C}$ and if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then show that,

i) $(a + c) \equiv (b + d) \pmod{n}$

ii) $(ax + cy) \equiv (bx + dy) \pmod{n}$

iii) $ac \equiv bd \pmod{n}$

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Solve the following system by Gauss elimination method:

$x+2y+z=2$

$2x-3y-4z=9$

$5z+4y+3z=8$

b) For any integer x, show that (a, b) = (a, b + ax).

c) Prove that $\left(1+i\sqrt{3}\right)^{-10}=2^{-11}\left(-1+i\sqrt{3}\right)$.

d) Given that $A=\begin{bmatrix} 4 & -1 & 3 \\ 0 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, find adj A and verify that A(adj A) = |A| I.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Find the modulus and argument of a complex number $z=\dfrac{3+i}{2-i}$.

b) Write in the form $x+iy$ of $\dfrac{(\cos\theta+i\sin\theta)^4}{(\cos\theta-i\sin\theta)^3}$.

c) Explain how to find the solution of non-homogenous system of linear equations of the form AX = B.

d) Find the eigen values of the matrix $A=\begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$.

e) Prove that if a|b and b|c then a|c.

f) Define : **i)** greatest common divisor (g.c.d).
ii) least common multiple (l.c.m).

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