**F.Y.B.SC. SEM – I (CBCS - 2016 Course) : SUMMER - 2019**
**SUBJECT: MATHEMATICS: ALGEBRA**

Day: Tuesday | Time: 11.00 A.M TO 02.00 PM
Date: 07/05/2019 | Max. Marks: 60

**S-2019-0808**

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**N.B.:**
1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

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**Q.1 A)** Select the correct alternatives. **(06)**

**i)** Rank of matrix $\begin{bmatrix} 4 & -1 & 0 \\ 2 & 3 & 6 \\ 2 & 3 & 6 \end{bmatrix}$ is --------.

a) 1    b) 3    c) 2    d) None of these

**ii)** If $A=\begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}$, then $A^{-1}=$-------.

$a)\ \frac{1}{4}\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$    $b)\ \frac{1}{4}\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}$

c) $\frac{1}{4}\begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}$    $d)\ -\frac{1}{4}\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}$

**iii)** If $z=x+iy$ then $|z|$ = ------.

$a)\ \sqrt{x^2+y^2}$    $b)\ \sqrt{x^2-y^2}$

$c)\ x^2+y^2$    $d)\ x^2-y^2$

**iv)** Product of complex number and its conjugate is -------.

a) complex number    b) Zero    c) real number    d) cannot judge

**v)** If $a|b$ and $b|a$ then $b=$------.

a) 1    b) 0    c) a    d) $\pm a$

**vi)** If $(a,b)=d$ then $\left(\frac{a}{d},\frac{b}{d}\right)=$-----.

a) d    b) 1    c) ad    d) bd

**B)** Answer the following questions: **(06)**

**i)** If $A=\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ then find $A^{-1}$.

**ii)** Define adjoint of square matrix.

**iii)** Find argument of complex number $Z=\frac{3+i}{2-i}$.

**iv)** Find the real and imaginary parts of $\frac{1-i}{1+i}$.

**v)** If $a\equiv b(\bmod n)$ then prove that $ax\equiv bx(\bmod n)$.

**vi)** Define g.c.d. of two integers.

**P. T. O.**

**Q.2** Attempt any **THREE** of the following: **(12)**

**a)** Let $a,b \in \mathbb{Z}$. Prove that if $a \mid b,\ b \neq 0$ then $|a| \leq |b|$. Further, if $a$ is proper divisor of $b$ then $|a| < |b|$.

**b)** Show that the following equations are consistent and solve it

$x+2y+3z=4$

$2x+2y+8z=7$

$x+y+9z=1$

**c)** Let $z_1, z_2 \in \mathbb{C}$ then show that:

$i)\ |z_1 z_2| = |z_1|\,|z_2|$ $\qquad ii)\ \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|},\ z_2 \neq 0.$

**d)** Find the eigenvalues of matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

**Q.3** Attempt any **FOUR** of the following: **(12)**

**a)** Find cube roots of unity.

**b)** If $A$ is a square matrix of order n then show that $(adj\ A)' = adj\ A'$.

**c)** Prove that $\left(1+i\sqrt{3}\right)^{-10} = 2^{-11}\left(-1+i\sqrt{3}\right)$.

**d)** Let $a$ and $b$ be non-zero integers and $d=(a,b)$. If $a=dx$ and $b=dy$ then show that $(x,y)=1$.

**e)** Solve the equation $x^9 - x^5 + x^4 - 1 = 0$ using De- Movire's theorem.

**Q.4** Attempt any **TWO** of the following: **(12)**

**a)** Prove that a necessary and sufficient condition for a square matrix $A$ to have the inverse is that $A$ is non-singular.

**b)** Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

**c)** Find non- singular matrices P and Q such that PAQ is normal form of matrix A and find rank of A, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 2 & 3 & 1 \end{bmatrix}$

**Q.5** Attempt any **TWO** of the following: **(12)**

**a)** Prove that if a and b are any two integers with $b \neq 0$ then there exist unique integers $q$ and $r$ such that $a = bq + r$, where $0 \leq r \leq |b|$.

**b)** Write the system of equations

$ax+by+cz=0,\ \ bx+cy+az=0,\ cx+az+bz=0$ as a matrix equation.

Show that the equations have a non-trivial solution if and only if $a+b+c=0$ or $a=b=c$.

**c)** Find g.c.d. of 3587 and 1819 and express it in the form 3587m +1819n, for some $m,n \in \mathbb{Z}$.

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