# F.Y.B.SC. SEM – I (2014 Course) : SUMMER - 2019
## SUBJECT: MATHEMATICS : ALGEBRA

Day : Wednesday
Date : 24/04/2019

**S-2019-0946**

Time : 12.00 NOON TO 02.00 PM
Max. Marks : 40

**N.B.:**

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

**Q.1** Attempt any **TWO** of the following: **(10)**

**a)** Show that if A is a non-singular matrix and $n \in N$ then $(A^n)^{-1} = (A^{-1})^n$.

**b)** Find the non-singular matrices P and Q such that PAQ is normal form and find rank A, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 2 & 3 & 1 \end{bmatrix}$$

**c)** Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Q.2** Attempt any **TWO** of the following: **(10)**

**a)** Prove that if $a$ and $b$ are any two integers with $b \neq 0$, then there exist unique integers $q$ and $r$ such that $a = bq + r$, where $0 \le r < |b|$.

**b)** Find g.c.d. of 3587 and 1819 and express it in the form 3587m + 1819 n. Find the values of m and n.

**c)** Test the following equations for consistency. If they are consistent find their general solution,

$x + y - 5z - u = 1, \quad 3x - y - 3z - 11u = -17,$

$4x + 5y - 23z - 2u = 9, \quad -x + y - z + 5u = 9$

**Q.3** Attempt any **TWO** of the following: **(10)**

**a)** State and prove De Moivre's theorem for positive and negative integers.

**b)** Prove that $(-1+i)^7 = -8(1+i)$

**c)** Solve the equation $x^9 - x^5 + x^4 - 1 = 0$, using De Moivre's theorem.

P.T.O.

**Q.4** Attempt any **FIVE** of the following: **(10)**

**a)** Find the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

**b)** Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

**c)** Show that if A is a non-singular matrix then $|A^{-1}| = \frac{1}{|A|}$.

**d)** Define congruence relation with suitable example.

**e)** Give the examples of the following:

**i)** A relation which is reflexive, transitive but not symmetric

**ii)** A relation which is symmetric but neither reflexive nor transitive.

**f)** Show that $\frac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^3} = \cos 7\theta + i\sin 7\theta$

**g)** Define g.c.d. of two integers 'a' and 'b'.

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