S.Y.B.SC. SEM – III (CBCS - 2016 Course) : SUMMER - 2019
SUBJECT : MATHEMATICS: CALCULUS OF SEVERAL VARIABLES

Day : Thursday
Date : 25/04/2019

S-2019-0835

Time : 03.00 P.M. To 06.00 P.M
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) Show that if f(x, y) is a function such that :
i) $f_x(a, b)$ and $f_y(a, b)$ exist and
ii) One of the first partial derivatives f_x, f_y is continuous at (a, b),
then f is differentiable at (a, b).

b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \text{ and } x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u \cos 3u.$$

c) If $u = f(x, y)$ and $x = r\cos\theta,\ y = r\sin\theta$

then show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) State and prove Taylor's theorem for a function of two variables x and y.

b) Find maxima or minima of $F(x, y) = x^4 + y^4 - (x + y)^2$.

c) Using Maclaurin's theorem show that

$$\sin x \sin y = xy - \frac{1}{6}\left[\left(x^3 + 3xy^2\right)\cos\theta x \sin\theta y + \left(y^3 + 3x^2 y\right)\sin\theta x \cos\theta y\right],$$

$0 < \theta < 1$.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) Explain Lagrange's method of undetermined multipliers.

b) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ by using Taylor's theorem.

c) Prove that $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ $\left(x^2 + y^2 + z^2 \neq 0\right)$, satisfies the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Change the order of integration and hence evaluate $\int_0^a \left[\int_y^a \frac{y\,dx}{\sqrt{x^2+y^2}} \right] dy.$

b) Find the area between the curves $x^2 = 4y$ and $x^2 = 8 - 4y$.

c) Evaluate $\iint_D (y-x)\,dx\,dy$ over the region D in the xy – plane bounded by the lines $y = x+1, y = x-3, y = -\frac{1}{3}x + \frac{7}{3}$ and $y = -\frac{1}{3}x + 5$.

d) Find by triple integration the volume of the solid bounded by $z = 0, x^2 + y^2 = 1$ and $x + y + z = 3$.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Examine the continuity of $f(x, y)$ at the origin where $f(x,y) = \frac{x+y}{x-y}$ for $x \neq y$ and $f(0, 0) = 0$.

b) If $u = \log\left(x^3 + y^3 - x^2y - xy^2\right)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{2}{x+y}$.

c) Show that $u = x^3 - 3xy^2$ is a harmonic function.

d) If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.

e) Change the order of integration of $\int_0^2 \left[\int_{2x}^{6-x} f\,dy \right] dx$.

f) Evaluate $\iint_R xy(x+y)\,dxdy$, where R is the rectangle $0 \le x \le 1,\ 0 \le y \le 1$.

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