S.Y. B. SC. (Computer Science) SEM –III (CBCS - 2016 COURSE) : SUMMER - 2019

SUBJECT : LINEAR ALGEBRA

Day : Tuesday
Date : 16/04/2019

S-2019-1090

Time : 03.00 PM TO 06.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) Let V be a vector space and W be its non-empty subset. Then W is a subspace of V if and only if it is closed under addition and scalar multiplication that is;

i) if $u, v \in W$ then $u + v \in W$

ii) if $u \in W$ and $k \in \mathbb{R}$ then $k\,u \in W$.

b) Show that the set $S = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \right\}$ is linearly independent in $M_{2\times 2}$.

c) Determine whether or not set S is basis for $V = \mathbb{R}^3$ where, $S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$.

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) Let $L : V \to W$ be a linear transformation then prove that kernel of L is a subspace of V.

b) Let $L : IR^3 \to IR^3$ be linear transformation defined by $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+c \\ a+b \\ b-c \end{bmatrix}$ then

i) find a basis for ker L.

ii) find a basis for range L.

c) Find standard matrix for linear transformation $T : IR^3 \to IR^4$ define as, $T(x, y, z) = (2x + 3y - z, x + y, -y + 2z, 2x)$.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) If λ is an eigen value of matrix A, then show that $\dfrac{\det(A)}{\lambda}$ is eigen value of adjoint of A i.e, adj A.

b) Find all the eigen values of A and find the eigen space corresponding to the smallest and largest eigen value of matrix

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}.$$

c) Find a matrix P that diagonalize the matrix $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Solve the following system of equations by Gauss elimination method.

$-x_2 - x_3 + x_4 = 0$

$x_1 + x_2 + x_3 + x_4 = 0$

$2x_1 + 4x_2 + x_3 - 2x_4 = -1$

$3x_1 + x_2 - 2x_3 + 2x_4 = 3$

b) For what value of a and b the system of linear equation:

$2x_1 - x_2 + 3x_3 = 2$

$x_1 + x_2 + 2x_3 = 2$

$5x_1 - x_2 + ax_3 = b$

have : **i)** No solution **ii)** Unique solution **iii)** Infinitely many solution.

c) Express the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ as a product of elementary matrices.

d) Find an LU factorization of the coefficient matrix of the given linear system $A\overline{X} = \overline{b}$ where, $A = \begin{bmatrix} 2 & 3 & 0 & 1 \\ 4 & 5 & 3 & 3 \\ -2 & -6 & 7 & 7 \\ 8 & 9 & 5 & 21 \end{bmatrix}$.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -1 & 5 \end{bmatrix}$. Find AB and BA. Is AB = BA?

b) Find all values of x so that $\overline{v}.\overline{v} = 1$ where $\overline{v} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ x \end{bmatrix}$.

c) Show that a vector space has only one zero vector.

d) Find the eigen values of matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$.

e) Define: **i)** Range of linear transformation.
ii) Kernel of linear transformation.

f) A map T : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by $T(x, y) = (xy, x - y)$. Determine whether T is a linear transformation?

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