F.Y. B. SC. (Computer Science) SEM –II (CBCS - 2016 COURSE) :
SUMMER - 2019
SUBJECT : ALGEBRA – II

Day : Monday Time : 03.00 PM TO 06.00 PM
Date : 15/04/2019 Max. Marks : 60

S-2019-1082

N. B. :

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 A) Choose the correct alternative: **(06)**

i) If a is any element in a group G then $(a^{-1})^{-1} =$ _____.
a) a **b)** a^{-1} **c)** 0 **d)** $(a^{-1})^{-1}$

ii) In any group G the identity element e is always of order _______.
a) 0 **b)** 1 **c)** 2 **d)** ∞

iii) In Z_6, $(\bar{8}+\bar{5}) =$ _____.
a) $\bar{0}$ **b)** $\bar{2}$ **c)** $\bar{1}$ **d)** $\bar{3}$

iv) S_n forms a group and order of S_n is _______.
a) n^2 **b)** $\frac{n}{n!}$ **c)** 0 **d)** $n!$

v) A homomorphic image of cyclic group is ______.
a) cyclic **b)** normal **c)** non cyclic **d)** none of these

vi) Every group of prime order is _______.
a) normal **b)** simple **c)** abelian **d)** none of these

B) Answer **ALL** the following questions: **(06)**

i) Prepare a composition table of $(Z_3, +_3)$.

ii) Define a group.

iii) Describe the subgroup $5Z \cap 4Z$ of a group $(Z, +)$.

iv) Express the following permutation as a product of disjoint cycles.

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 7 & 6 & 8 & 1 \end{pmatrix}$$

v) Define an isomorphism.

vi) Define an integral domain.

P.T.O.

Q.2 Attempt any **THREE** of the following questions: **(12)**

a) Prove that intersection of two subgroups of a group is a subgroup.

b) Show that $(Z_7{}^*, +_7)$ is a group.

c) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 \end{pmatrix}$ as a product of disjoint cycles. Determine whether σ is even or odd. Find σ^{-1}.

d) Find the order of every element in $(Z_6, +_6)$.

Q.3 Attempt any **FOUR** of the following: **(12)**

a) Prove that every cyclic group is abelian.

b) Compute the indicated product of cycle that is permutation on $\{1,2,3,4,5,6\}$
$\sigma = (1\ 3\ 5)\ (2\ 4\ 6)\ (1\ 2)$.

c) Show that $(Z,+)$ is isomorphic to $(mZ,+)$.

d) Prove that every subgroup of an abelian group is normal.

e) Show that a group G is abelian if $b^{-1}a^{-1}ba = e \quad \forall a,b \in G$.

Q.4 Attempt any **TWO** of the following: **(12)**

a) A subgroup H of a group G is normal if and only if $xHx^{-1} = H \quad \forall x \in G$.

b) State and prove Lagrange's theorem.

c) Let G be a group and $a,b,c \in G$ then prove that :
i) Left cancellation law: $a \cdot b = a \cdot c \Rightarrow b = c$
ii) Right cancellation law: $b \cdot a = c \cdot a \Rightarrow b = c$

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Let G be a group and $a,b \in G$ then prove that $o(a) = o(b^{-1}ab)$.

b) Find all the subgroups of a cyclic group $G = \{a, a^2, a^3, ...a^{30} = e\}$.

c) Given : $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$
Compute : **i)** fog **ii)** (fog)oh **iii)** $h^{-1}og^{-1}$

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