F.Y. B. SC. (Computer Science) SEM – II (CBCS 2018 COURSE)

SUMMER - 2019

SUBJECT : ALGEBRA - II

Day : Saturday **Time :** 11.00 AM TO 02.00 PM

Date : 27/04/2019 **S-2019-1066** **Max. Marks : 60**

N.B.

1) All questions are **COMPULSORY**.
2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) Let G be a group then prove that :
i) Identity element in G is unique
ii) Every element of G has unique inverse.

b) Let G = $\mathbb{Z}$ be the set of integers. Define $*$ as, for a,b $\in \mathbb{Z}$ $a*b = a+b-2$.
Show that (G, $*$) is an abelian group.

c) Given : $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$

Find : i) f^{-1} ii) g^{-1} iii) h^{-1} iv) fog v) gof^{-1} vi) $h^{-1}og^{-1}$

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) Show that the intersection of two subgroups of a group is a subgroup. Whether the union of two subgroups is a subgroup again ? Justify.

b) State and prove Lagrange's theorem.

c) Find all the subgroups of a cyclic group of order 12. Draw Hassse diagram for these subgroup relation.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) The subgroup H of a group G is a normal subgroup of G if and only if each left coset of H is a right coset of H in G.

b) Let G be the group of 2×2 invertible matrices under matrix multiplication $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G$ if $ad-bc\neq0$.

Show that $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} / a\neq0 \right\}$ is a normal subgroup of G.

c) Show that $(\mathbb{Z},+)$ is isomorphic to $(m\mathbb{Z},+)$.

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) Let $f:(\mathbb{Z},+)\rightarrow(\mathbb{R},+)$ be defined by $f(n) = 5n \ \forall n$. Test whether f is homomorphism. If so, find its Kernel.

b) Find the order of every element in $(\mathbb{Z}_6,+_6)$.

c) Prove that every cyclic group is abelian.

d) Write down factor group $\frac{\mathbb{Z}_6}{<2>}$.

P.T.O.

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) Prepare a multiplicative table for a group G = {1, –1, i, –i}.

b) Prove that every subgroup of an abelian group is normal.

c) Let G be a group a, b, c $\in$ G then prove that if $a \cdot b = a \cdot c \Rightarrow b = c$.

d) Find the following subgroups of group $(\mathbb{Z}, +)$
i) $5\mathbb{Z} \cap 4\mathbb{Z}$ ii) the only finite subgroup of $\mathbb{Z}$.

e) Find the order of permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 2 & 5 & 4 & 1 \end{pmatrix}$.

f) Define : **i)** Ring **ii)** Field

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