F.Y. B. SC. (Computer Science) SEM – I (CBCS 2018 COURSE) : SUMMER - 2019

SUBJECT : ALGEBRA – I

Day : Monday
Date : 15/04/2019

S-2019-1054

Time : 03.00 PM To 06.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) Prove that R is equivalence relation, if R be a relation of $\mathbb{Z}$ define by xRy if and only if 5x + 6y is divisible by 11 for x, y $\in \mathbb{R}$.

b) Find the matrix of transitive closure for $M(R)=\begin{bmatrix}1&0&0&1\\1&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ using Warshall's algorithm.

c) If a, b, c, d $\in \mathbb{Z}$, n $\in \mathbb{N}$ and a $\equiv$ b (mod n), c $\equiv$ d (mod n) then prove that

i) (a + c) $\equiv$ (b + d) (mod n)
ii) (a – c) $\equiv$ (b – d) (mod n)
iii) ac $\equiv$ bd (mod n)

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) If $z_1, z_2 \in \mathbb{C}$ then prove that $\left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|$ and $\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$.

b) Solve the equation $x^7 + 1 = 0$.

c) State De Moivre's theorem and use it to prove
$\cos 4\theta = \cos^4\theta - 6\cos^2\theta \sin^2\theta + \sin^4\theta$
$\sin 4\theta = 4\cos^3\theta \sin\theta - 4\cos\theta \sin^3\theta$.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) State and prove Euclid's Lemma.

b) Find the g.c.d. of 3927 and 377 and express the g.c.d in the form 3927m + 377n.

c) Construct a decoding table with syndromes for a group code given by generator matrix $G=\begin{bmatrix}1&0&1&1&0\\0&1&0&1&1\end{bmatrix}$. Use the table to decode the received word 11110.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Let $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x^2 - 1$,

$g : \mathbb{R} \to \mathbb{R}$ such that $g(x) = \dfrac{3x-4}{10}$. Obtain gof (x) and fog (x).

b) If (a, m) = (b, m) = 1 then show that (ab, m) = 1.

c) Find the loci of point z satisfying the relation $|z - 2| = 2\,|z - 1|$.

d) Find all the code words of the code determined by the parity check matrix,

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Find the modulus and argument of $z = \dfrac{3-i}{2+i} + \dfrac{3+i}{2-i}$.

b) Prove that $(-1+i)^7 = -8(1+i)$.

c) Draw the directed graph of relation R on a set A = {a, b, c, d, e} is defined as R = {(a, a), (a, b), (b, c), (b, d), (c, d), (c, e), (d, b), (d, c), (e, a)}.

d) Obtain the remainder when $4^{37} + 82$ is divided by 7.

e) Find the minimum distance d for the following code:
C = {1101, 1001, 0110, 1110} in $\mathbb{Z}_2^4$.

f) Define : **i)** greatest common divisor (g.c.d)
ii) least common multiple (l.c.m)

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